

Spaces of Discrete Time Trees

Lena Collienne

Matsen Group



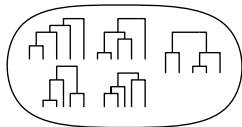
Fred Hutch
Cancer Center

July 18th, 2023

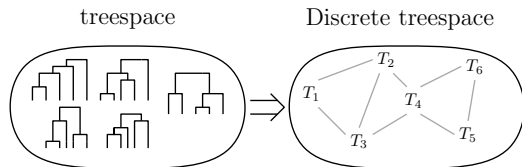
Overview

Overview

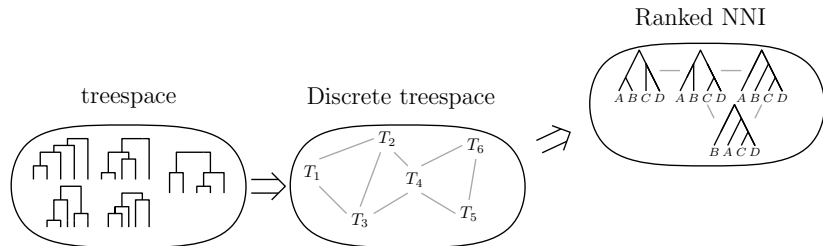
treespace



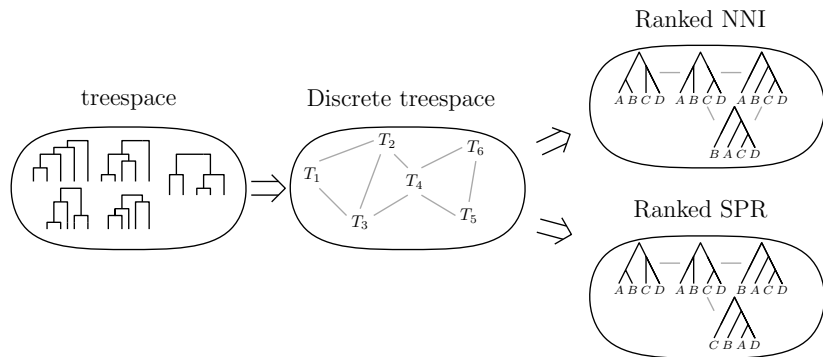
Overview



Overview

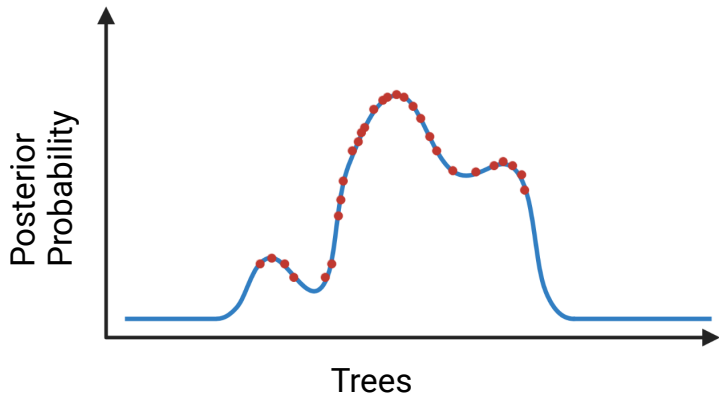


Overview



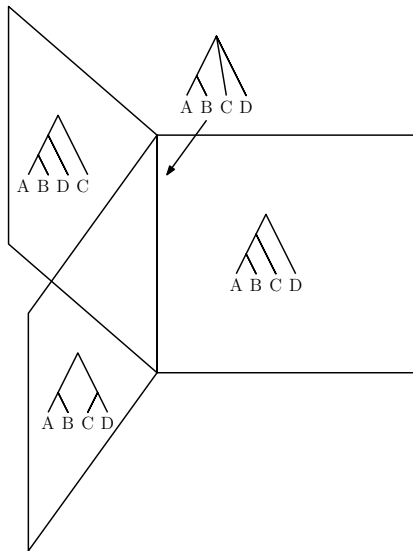
Tree Inference

Bayesian Inference



Treespace

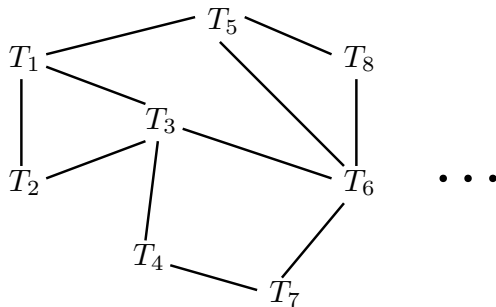
BHV-space



[after Billera, Holmes, and Vogtmann, 2001]

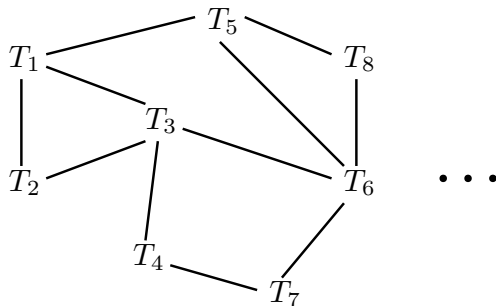
Discrete Treespace

Tree rearrangements



Discrete Treespace

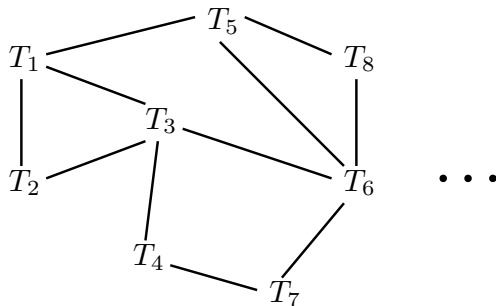
Tree rearrangements



- ▶ tree rearrangements: NNI, SPR, TBR

Discrete Treespace

Tree rearrangements



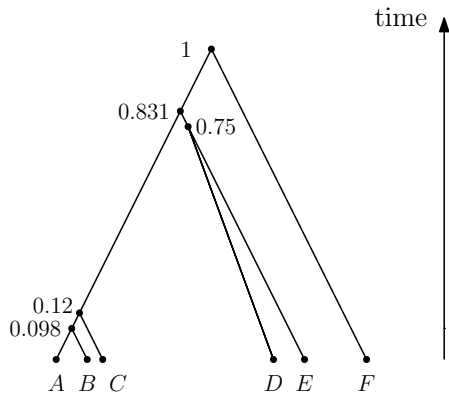
- ▶ tree rearrangements: NNI, SPR, TBR $\rightarrow \mathcal{NP}$ -hard

Discrete Treespace

Ranked trees

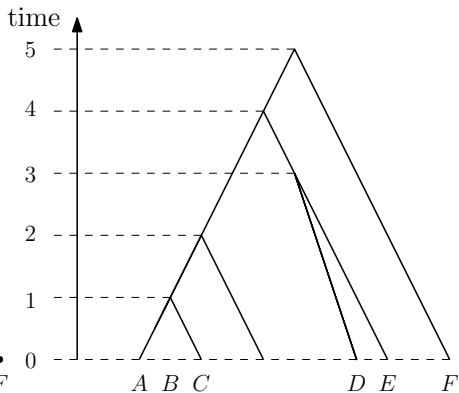
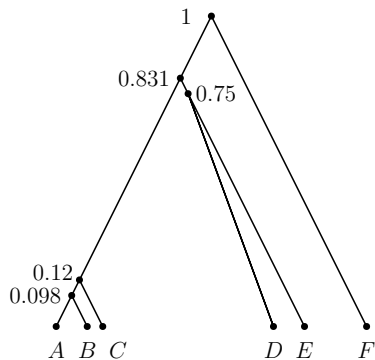
Discrete Treespace

Ranked trees



Discrete Treespace

Ranked trees



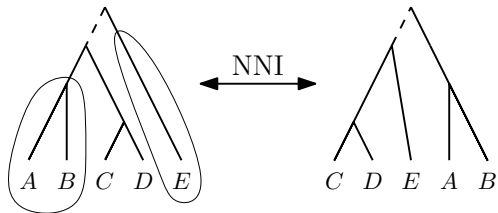
NNI on Ranked Trees

NNI moves



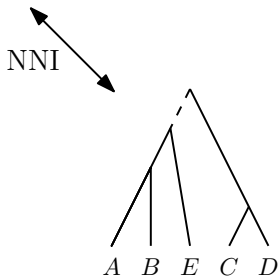
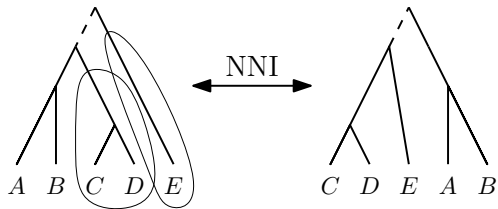
NNI on Ranked Trees

NNI moves



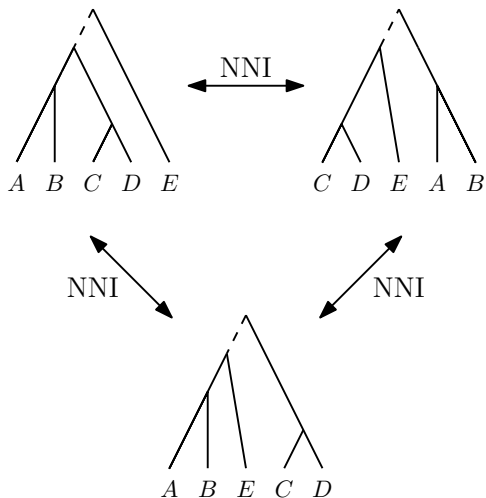
NNI on Ranked Trees

NNI moves



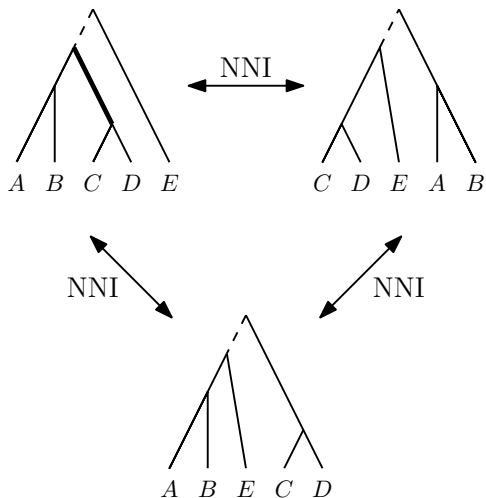
NNI on Ranked Trees

NNI moves



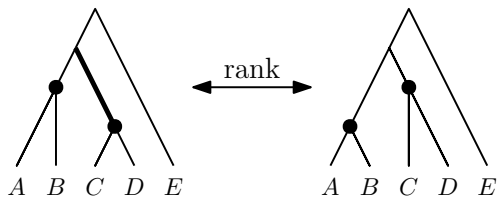
NNI on Ranked Trees

NNI moves

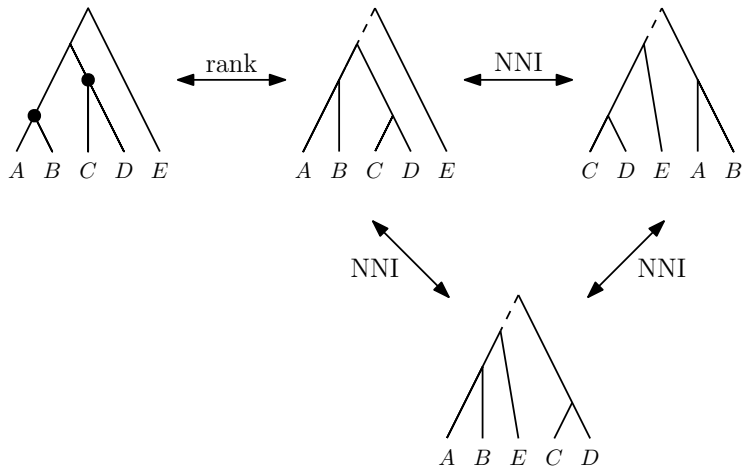


NNI on Ranked Trees

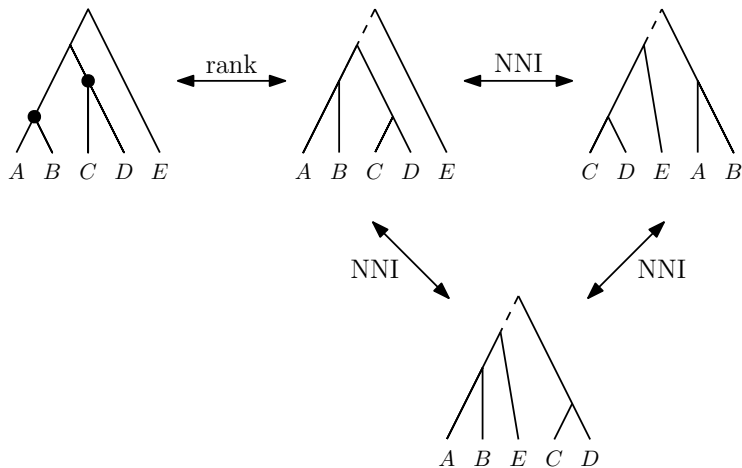
Rank move



RNNI



RNNI



Theorem

Shortest paths in RNNI can be computed in $O(n^2)$.

RNNI

Probability Distributions



Lars Berling

RNNI

Probability Distributions

- ▶ Simulate tree + alignment
- ▶ Run BEAST2 and log every tree \rightarrow 1,000 trees



Lars Berling

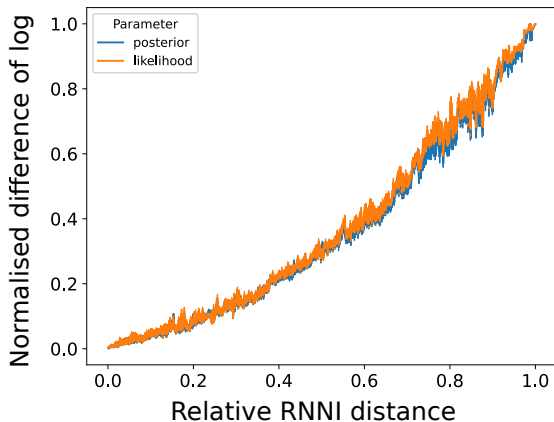
RNNI

Probability Distributions



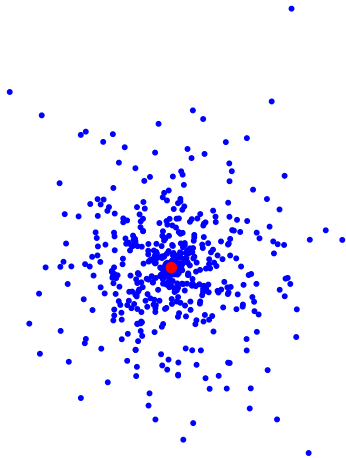
Lars Berling

- ▶ Simulate tree + alignment
- ▶ Run BEAST2 and log every tree \rightarrow 1,000 trees



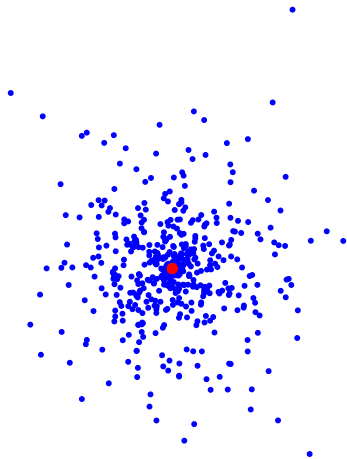
RNNI

Mean trees



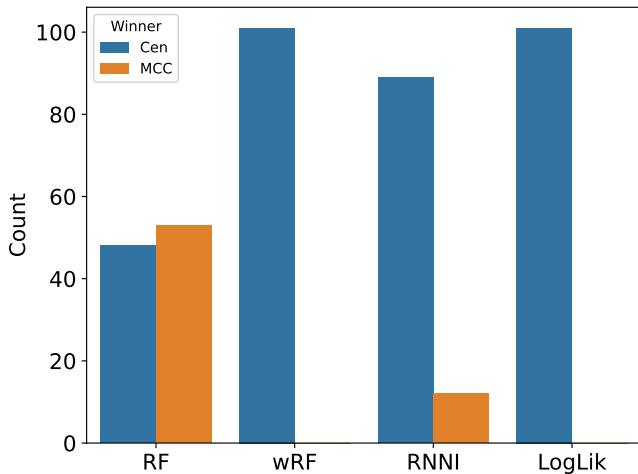
RNNI

Mean trees



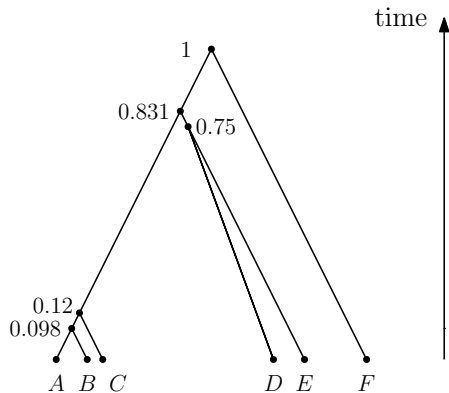
$$\text{Cen}(\mathcal{T}) = \arg \min_{T^*} \sum_{T \in \mathcal{T}} d(T, T^*)^2$$

Simulation study + BEAST2

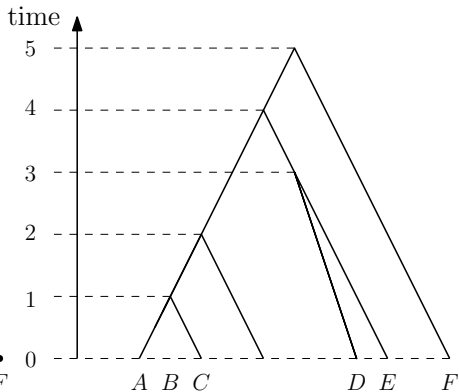
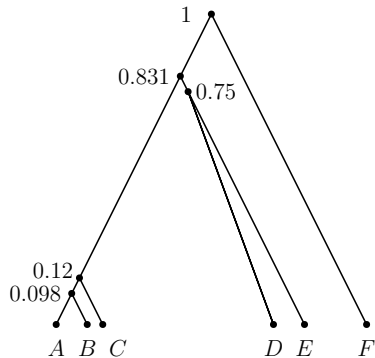


Generalising RNNI

Generalising RNNI

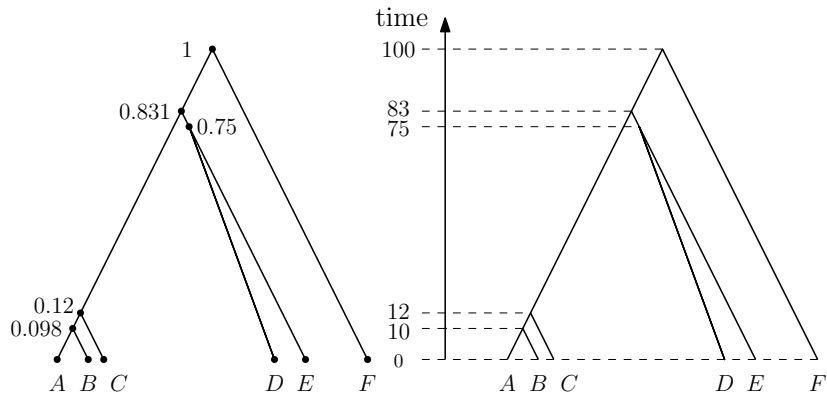


Generalising RNNI



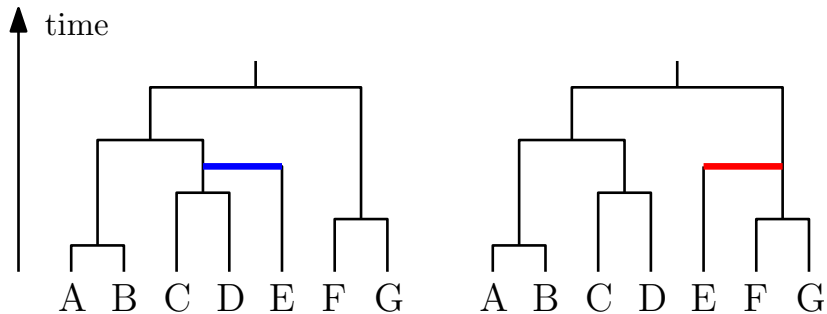
Generalising RNNI

Discrete Coalescent Trees



Subtree Prune and Regraft

Subtree Prune and Regraft

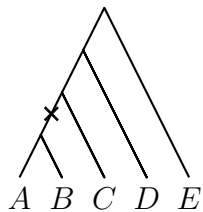


SPR on Ranked Trees

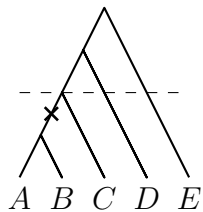
SPR on Ranked Trees



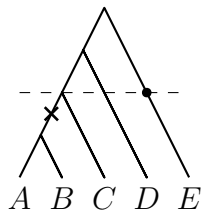
SPR on Ranked Trees



SPR on Ranked Trees

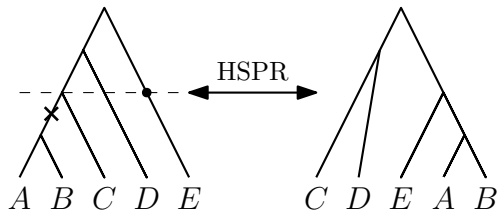


SPR on Ranked Trees



SPR on Ranked Trees

Horizontal SPR



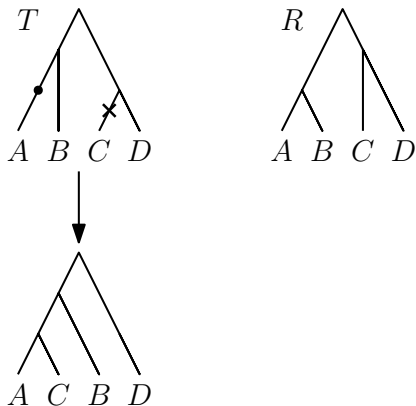
SPR on Ranked Trees

HSPR is connected



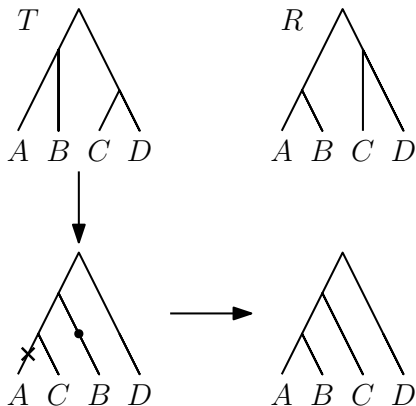
SPR on Ranked Trees

HSPR is connected



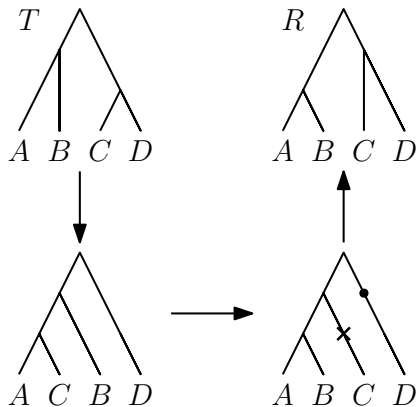
SPR on Ranked Trees

HSPR is connected



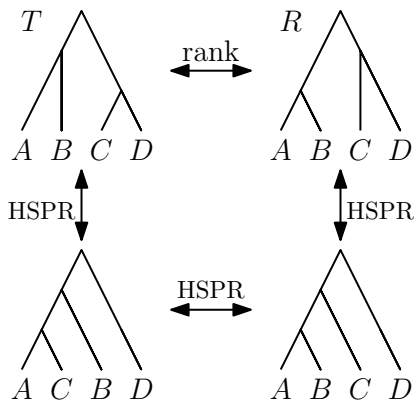
SPR on Ranked Trees

HSPR is connected



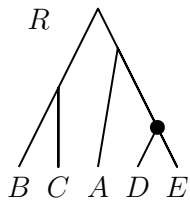
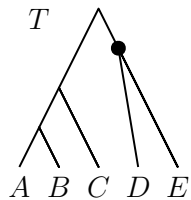
SPR on Ranked Trees

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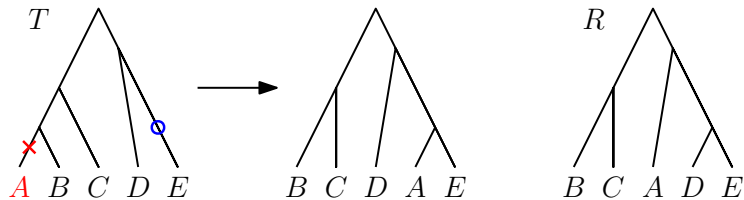
SPR on Ranked Trees

Clusters



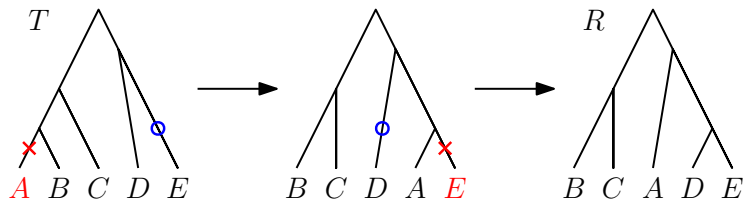
SPR on Ranked Trees

Clusters



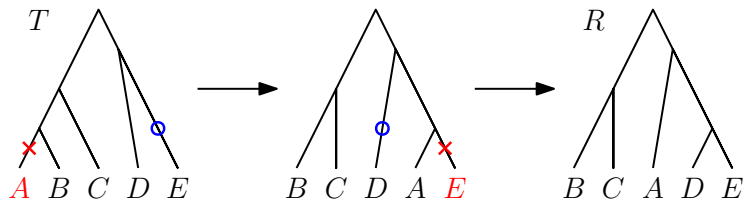
SPR on Ranked Trees

Clusters



SPR on Ranked Trees

Clusters



Open Question: What is the complexity of computing distances?

SPR on Ranked Trees

Adding leaves

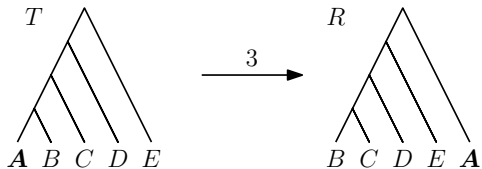
SPR on Ranked Trees

Adding leaves



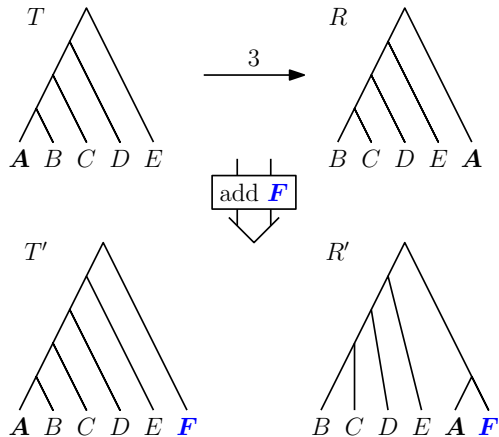
SPR on Ranked Trees

Adding leaves



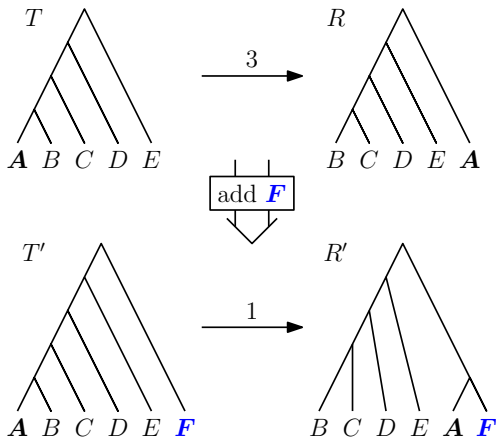
SPR on Ranked Trees

Adding leaves



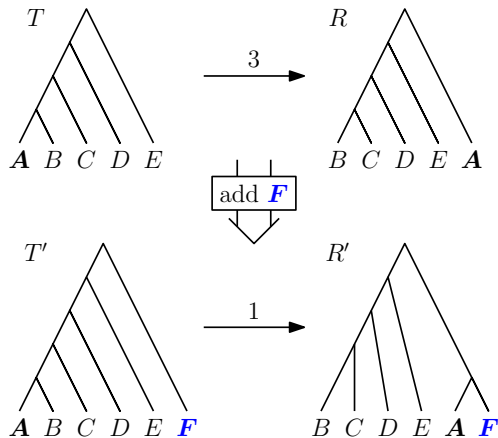
SPR on Ranked Trees

Adding leaves



SPR on Ranked Trees

Adding leaves



Theorem

Adding a leaf to a tree can decrease the distance by $O(n)$.

Thank you



- ▶ Alex Gavryushkin
- ▶ Lars Berling



- ▶ David Bryant



- ▶ Chris Whidden



Supplement

Probability distributions in BHV-space

