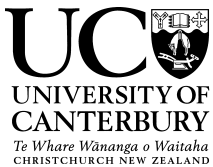
Subtree Prune and Regraft on Ranked Trees

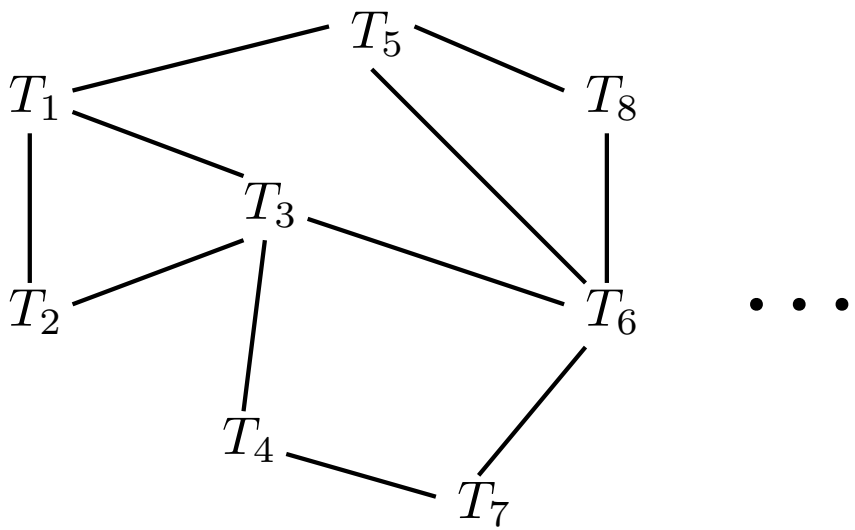
Lena Collienne



Biological Data Science Lab
University of Canterbury/Otago

24/11/2022

Discrete Tree Space



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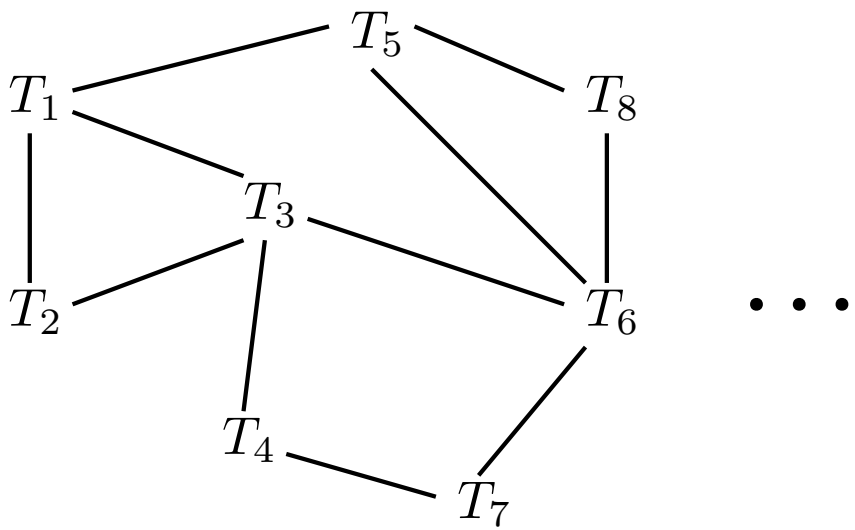

Popular tree rearrangement operations: NNI, SPR, TBR

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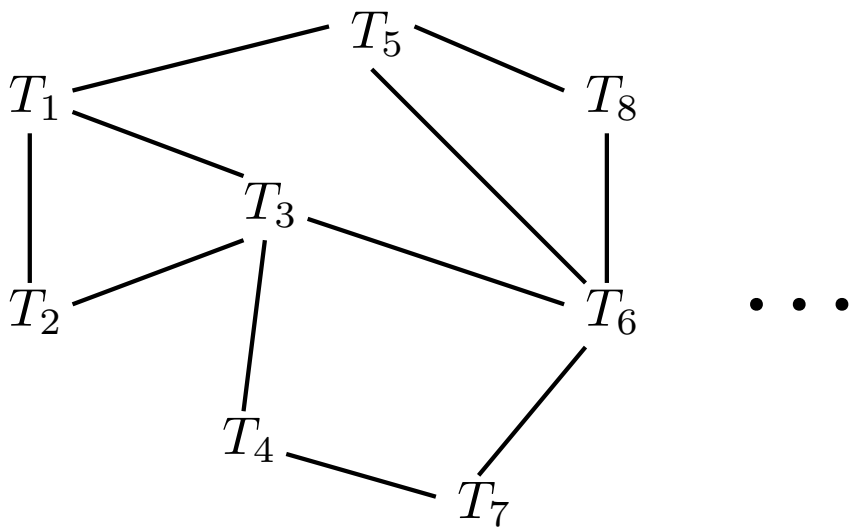
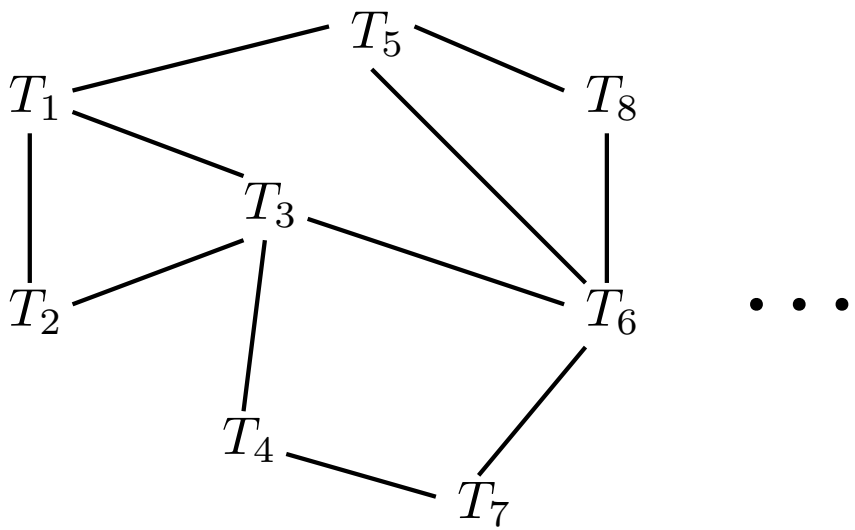


Popular tree rearrangement operations: NNI, SPR, TBR

- Similarity measure

Discrete Tree Space



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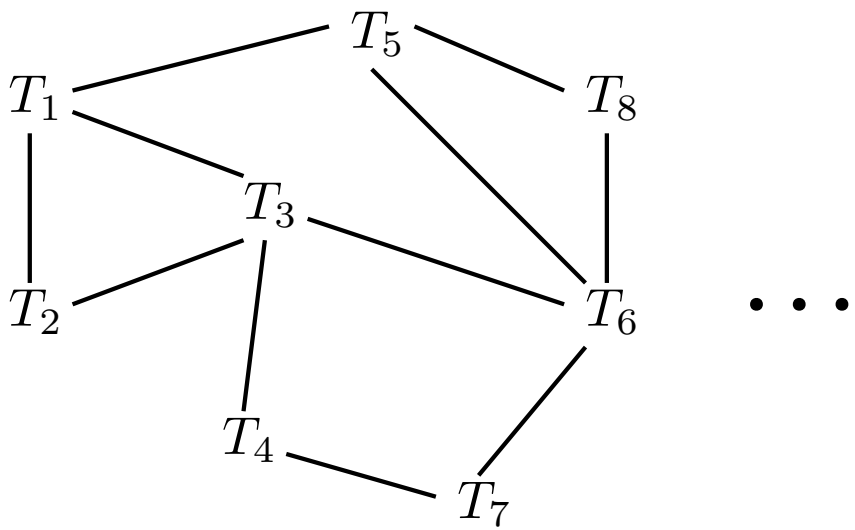
Discrete Tree Space



Popular tree rearrangement operations: NNI, SPR, TBR

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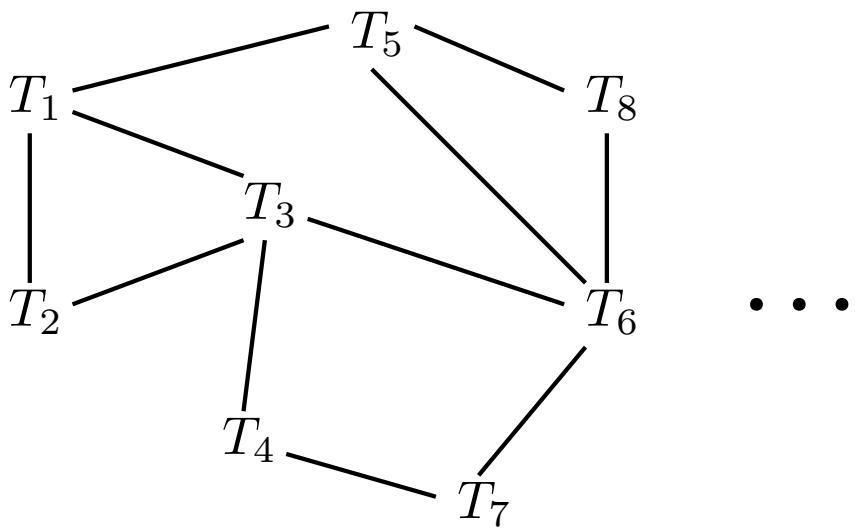
Problem: Computing distances is $\mathcal{NP}$-hard

Discrete Tree Space



Popular tree rearrangement operations: NNI, SPR, TBR

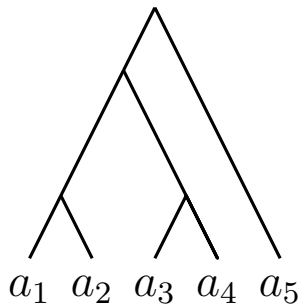
- Similarity measure
- Statistics on trees

Problem: Computing distances is $\mathcal{NP}$-hard

Exception: RNNI

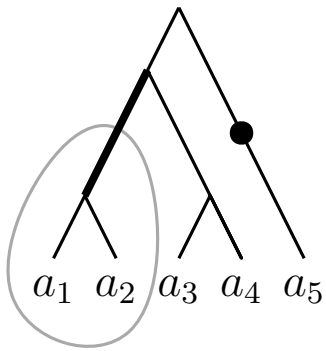
SPR

Subtree Prune and Regraft



SPR

Subtree Prune and Regraft



SPR

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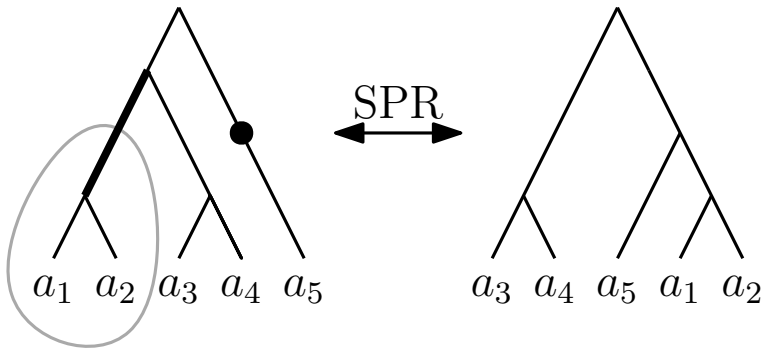
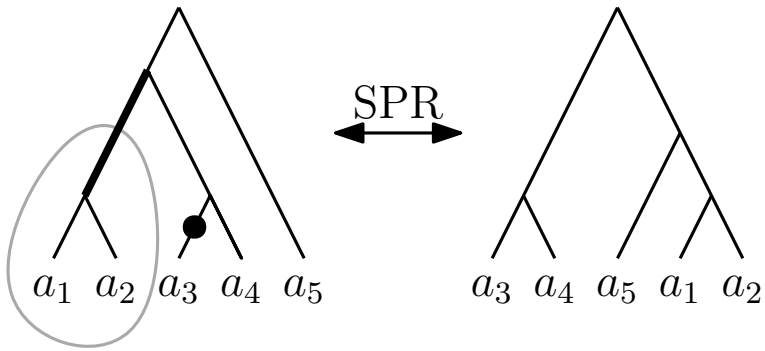


SPR

Subtree Prune and Regraft



SPR

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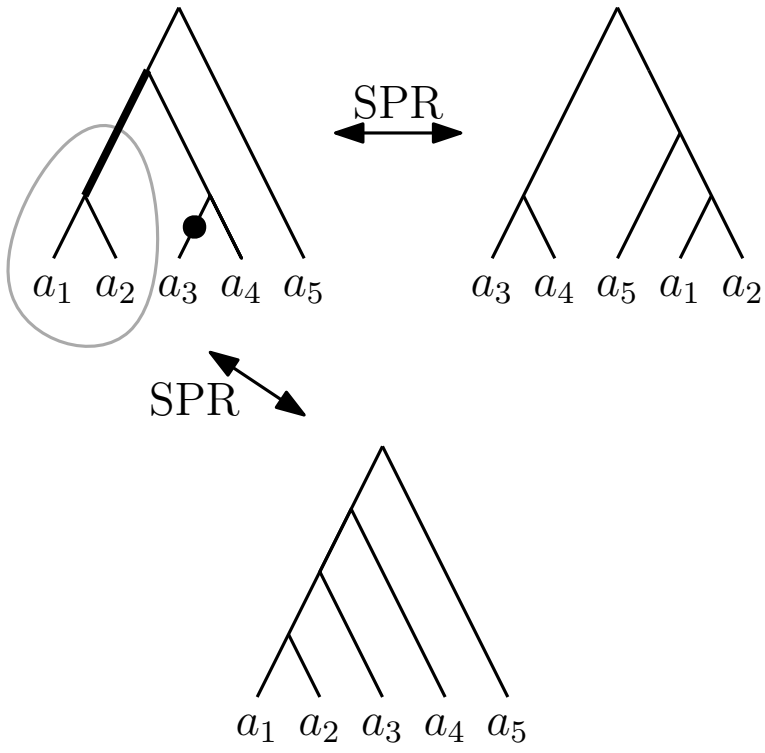

SPR

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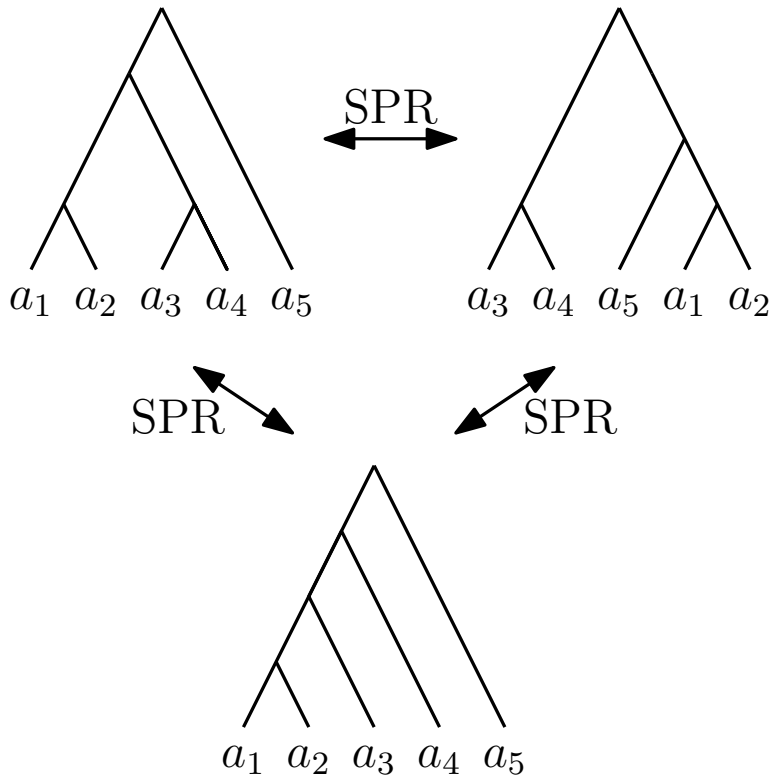
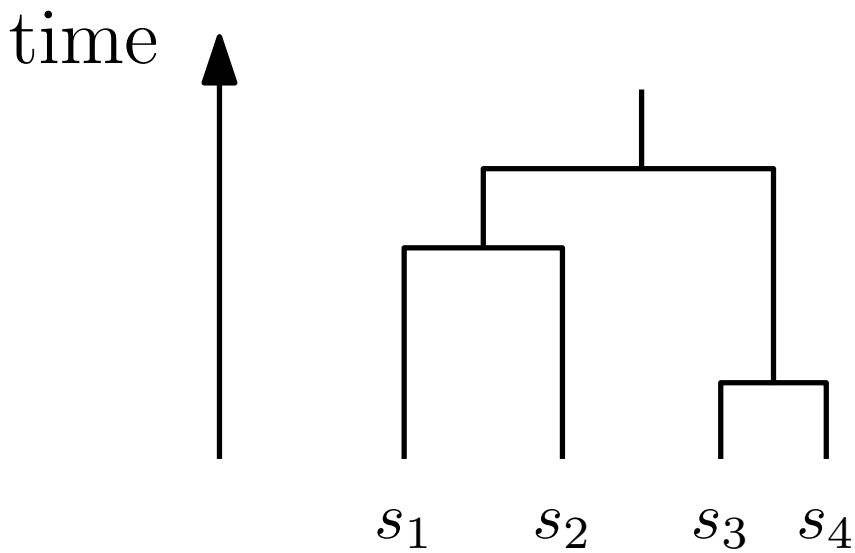
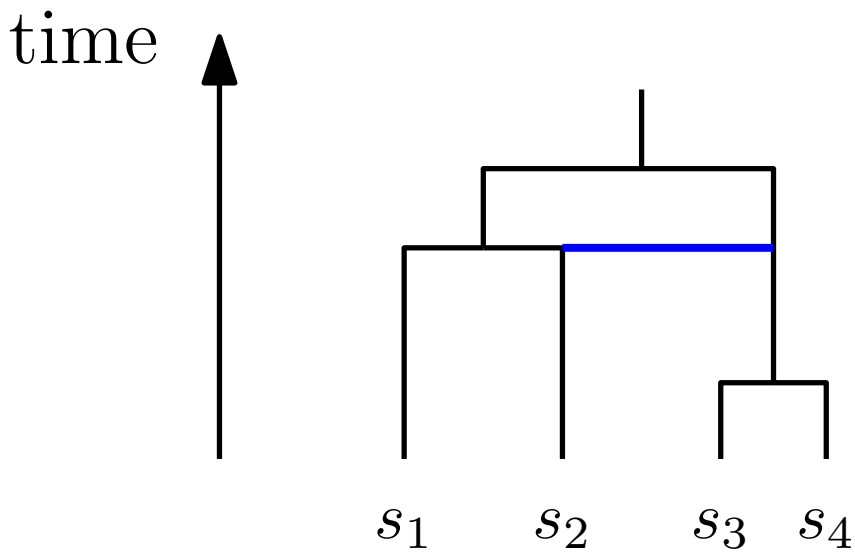


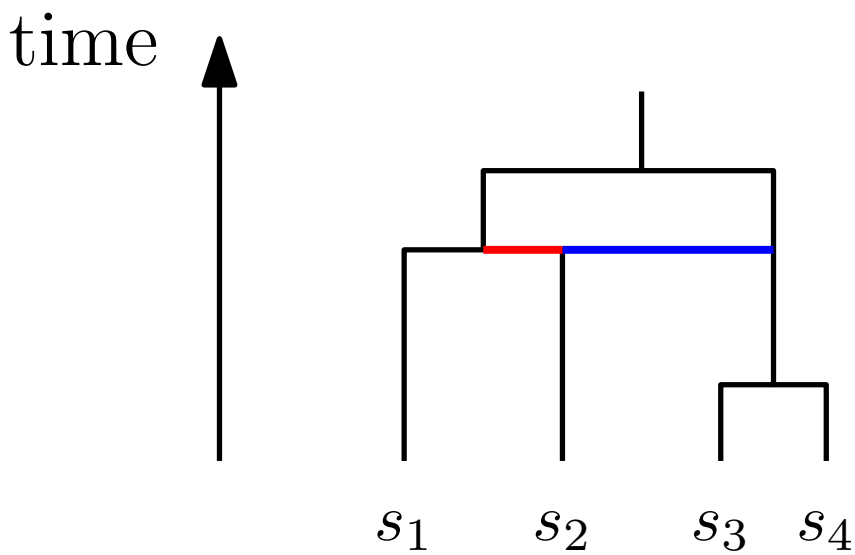
Time trees



Time trees

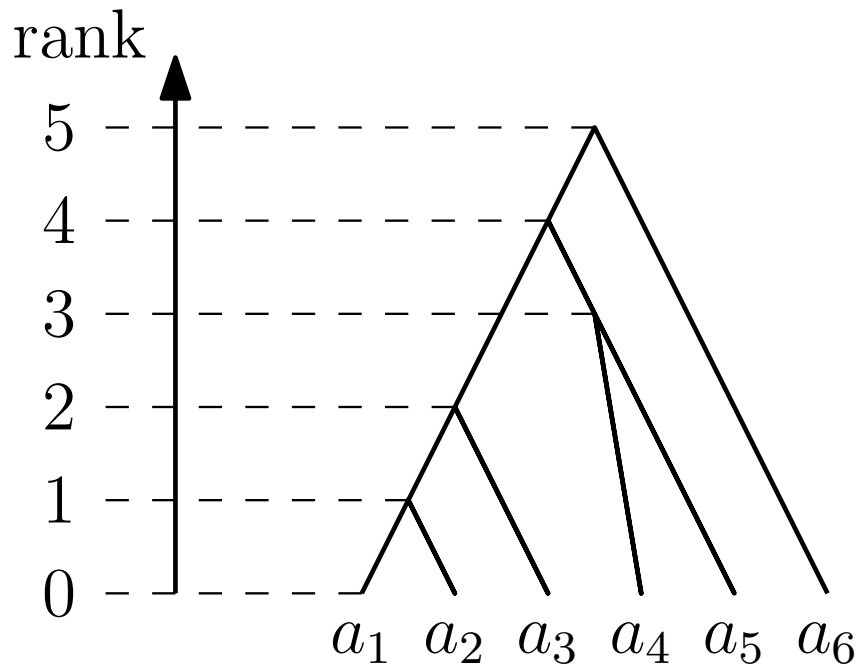


Time trees



Time trees

Ranked Trees



HSPR – Horizontal Subtree Prune and Regraft

HSPR move

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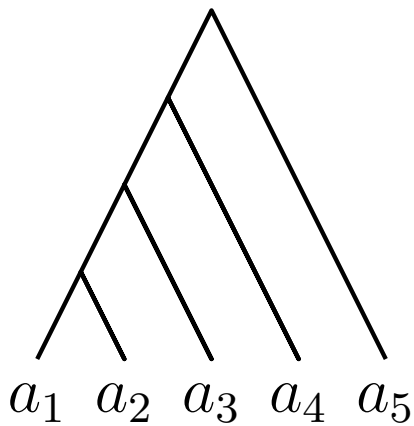
HSPR move



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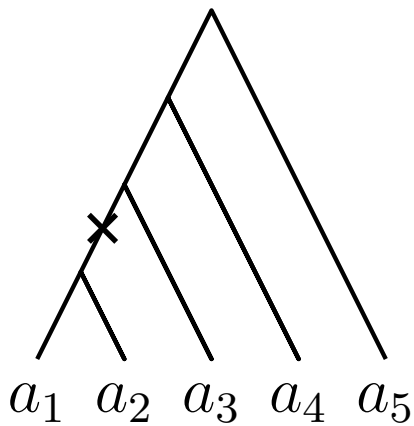

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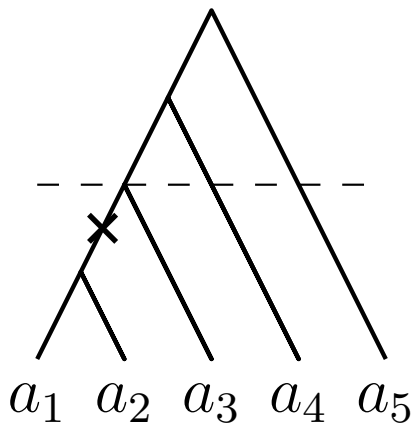

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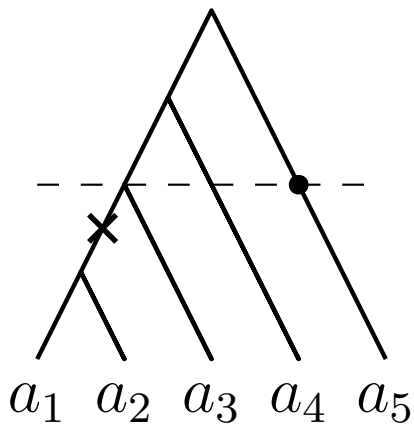

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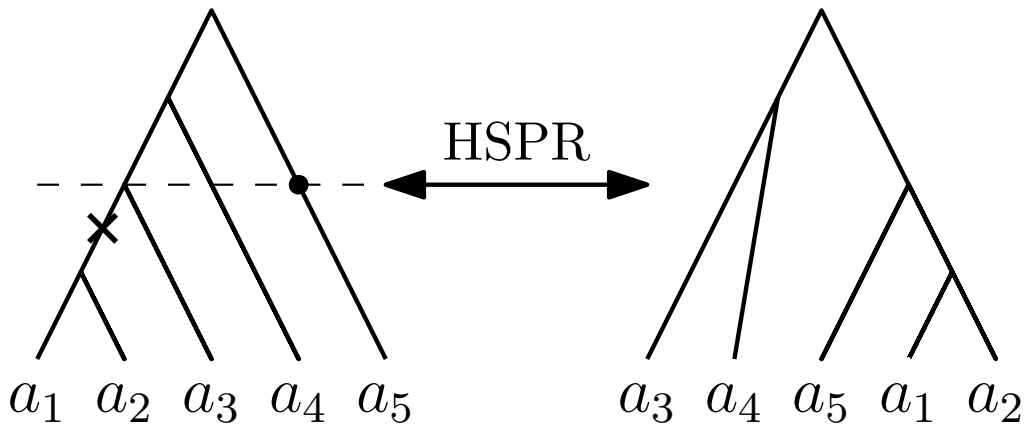
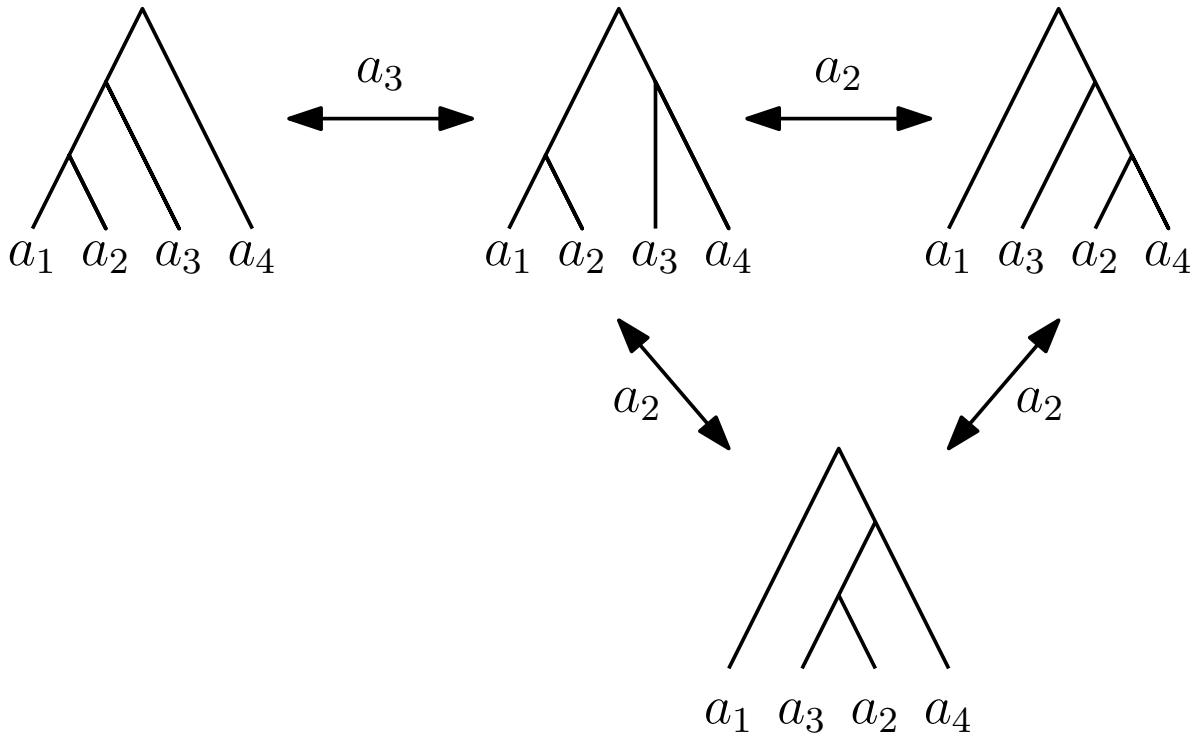
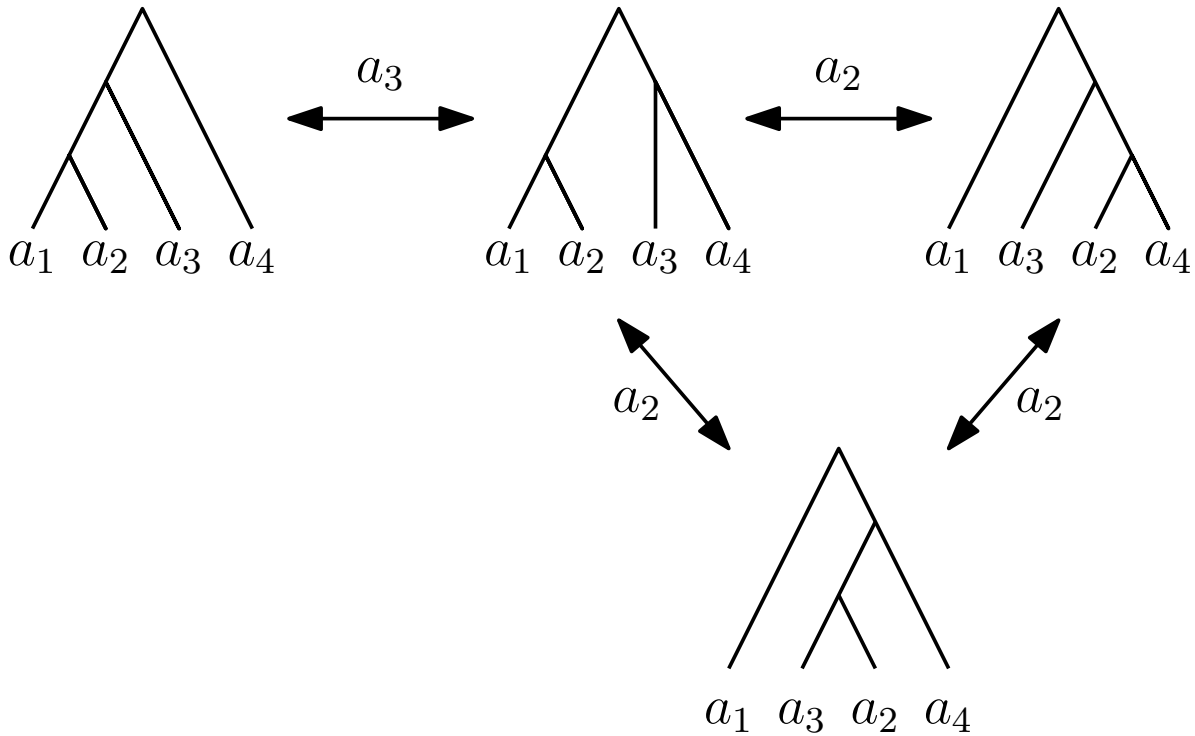


HSPR space



HSPR space

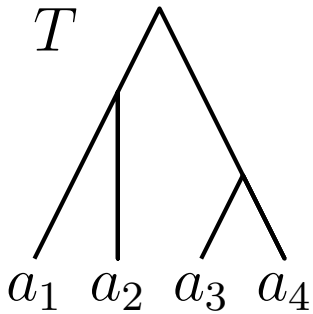


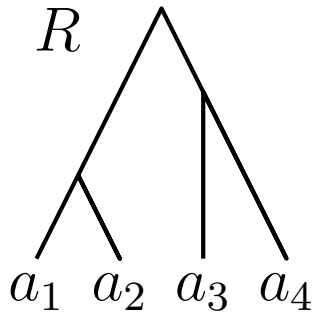
Theorem

The HSPR *space is connected.*

HSPR space

What if just ranking changes?





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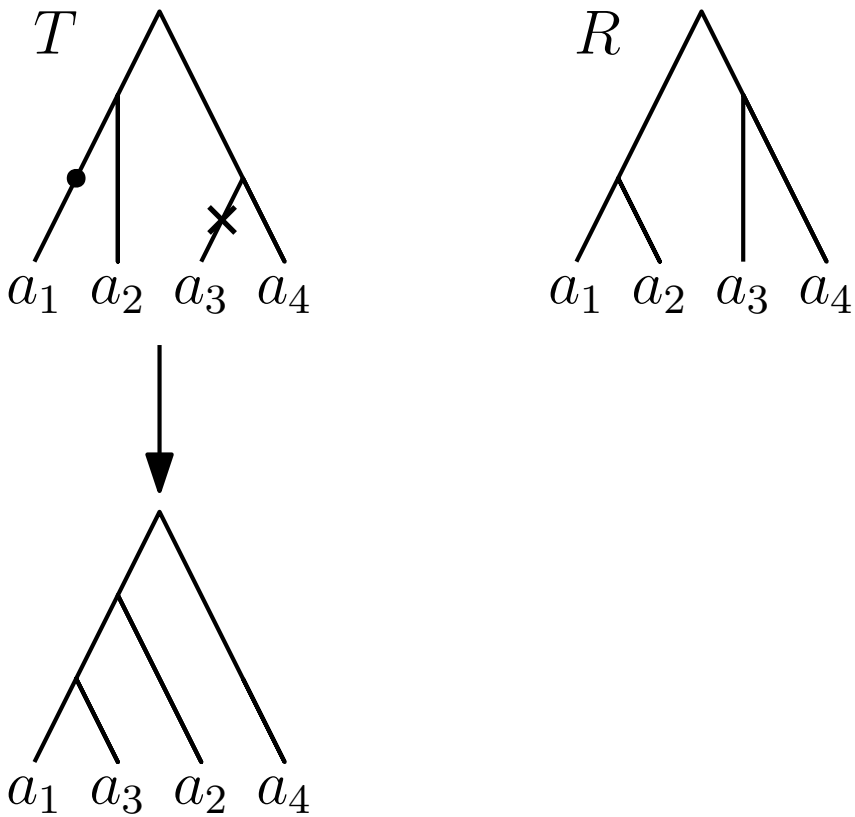

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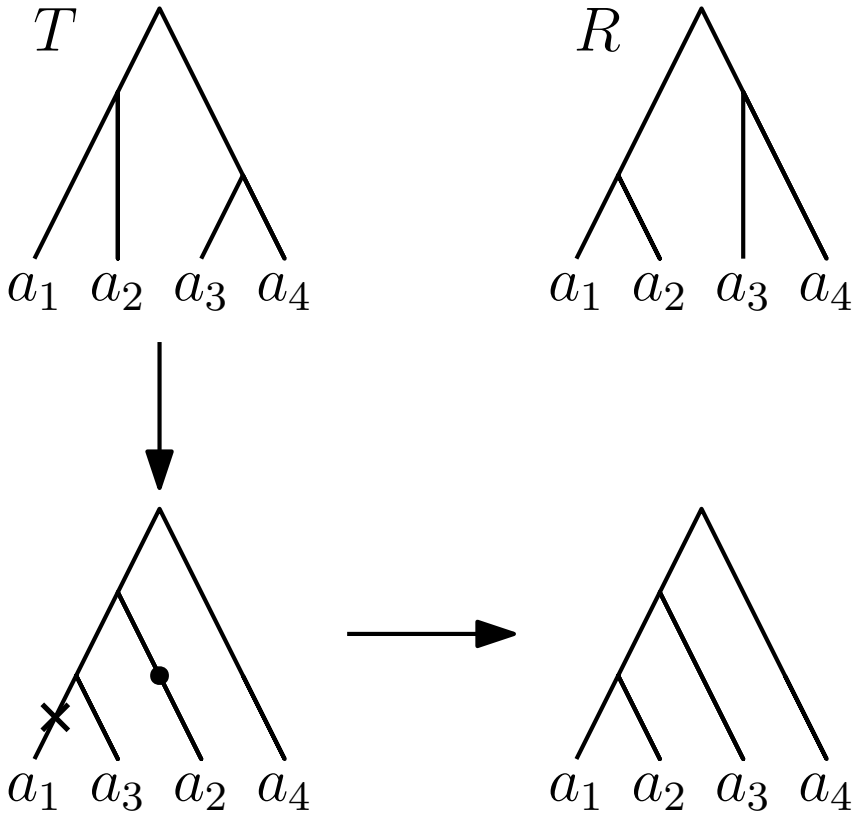

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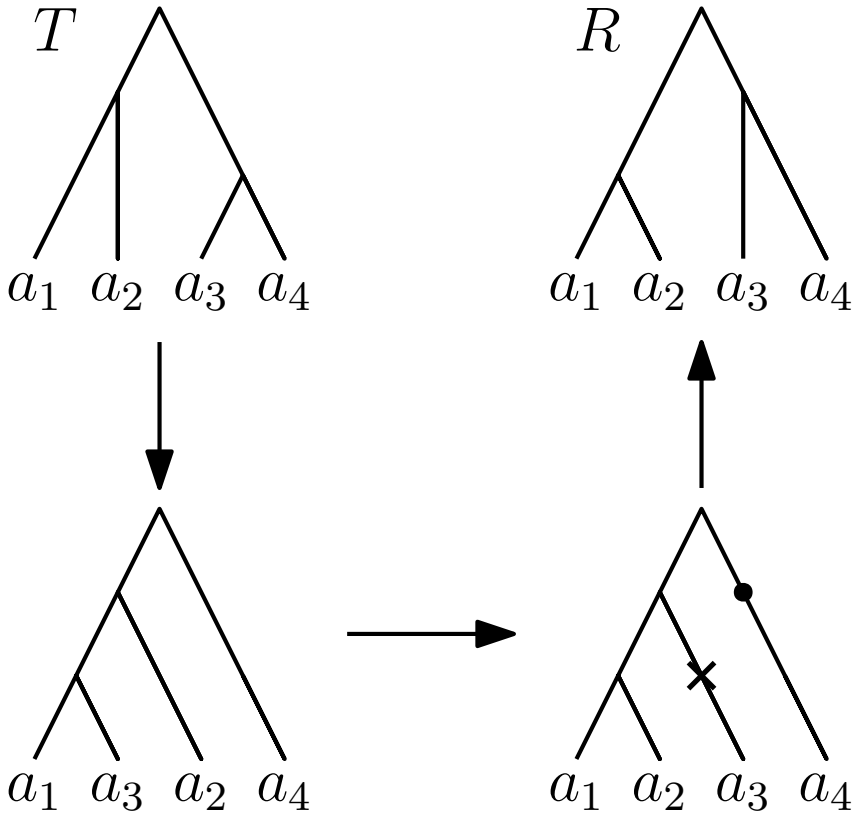
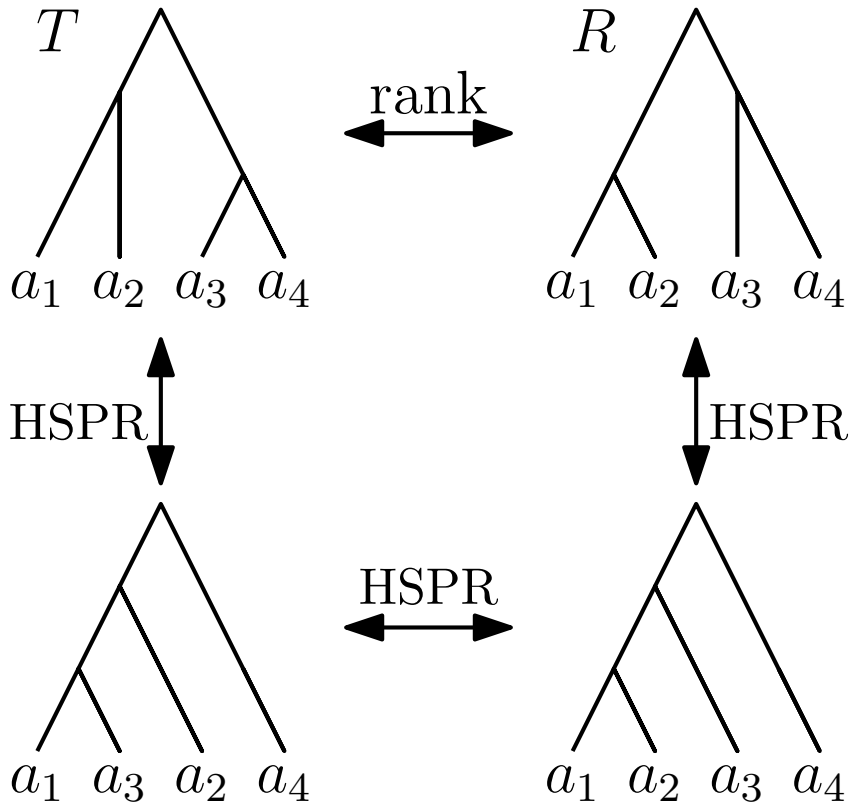


RSPR


T
R
rank
a_1 a_2 a_3 a_4
a_1 a_2 a_3 a_4
HSPR
HSPR
HSPR
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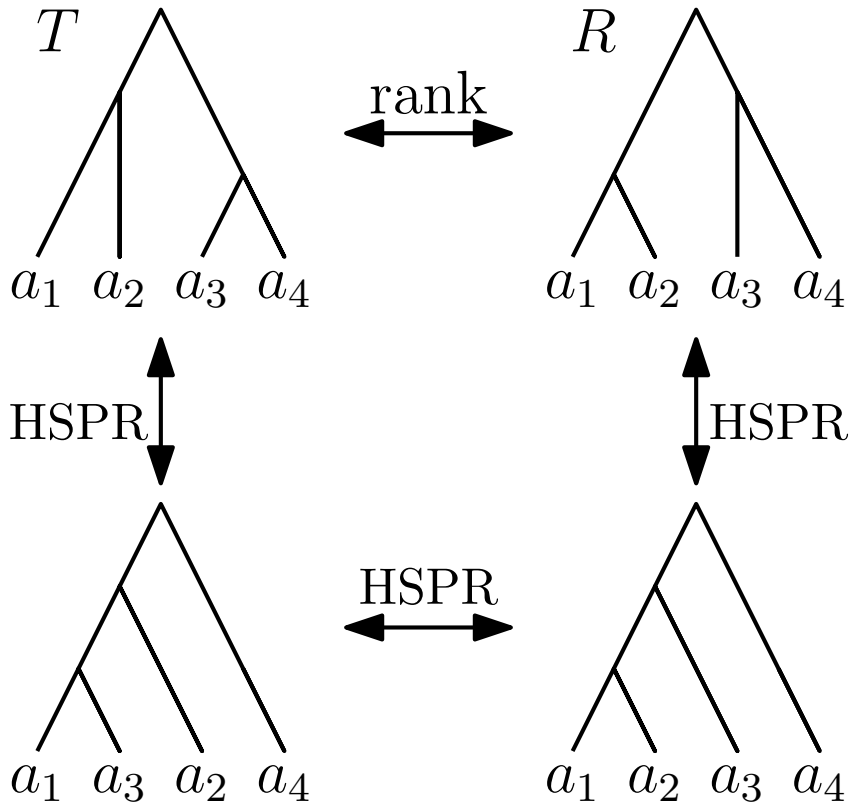
a_1 a_2 a_3 a_4

RSPR



Theorem

There always is a shortest RSPR *path with first rank moves, then* HSPR *moves.*

HSPR space

$n = 5$

HSPR space

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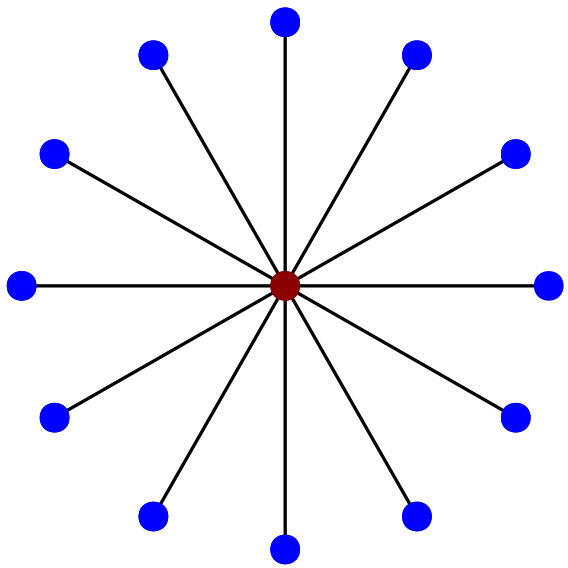
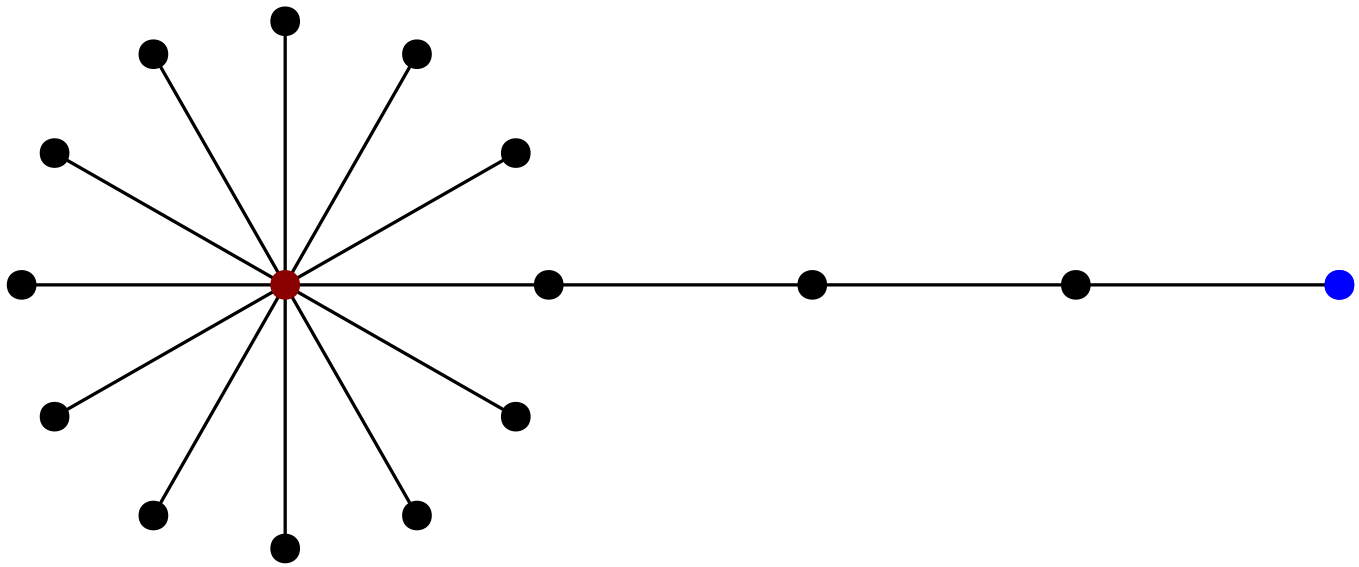


- Neighbourhood size: $(n-1)(n-2)$

HSPR space

$n = 5$



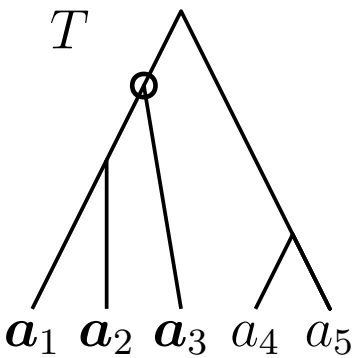
- Neighbourhood size: $(n-1)(n-2)$
- Diameter: $\lfloor \frac{3}{2}(n-2) \rfloor$ (conjecture)

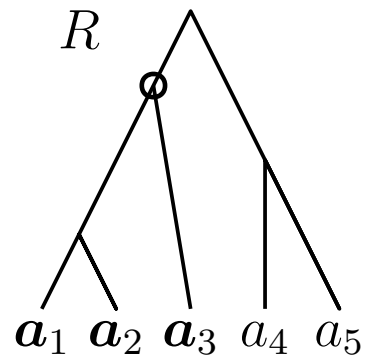
Shortest paths

Clusters

Shortest paths

Clusters

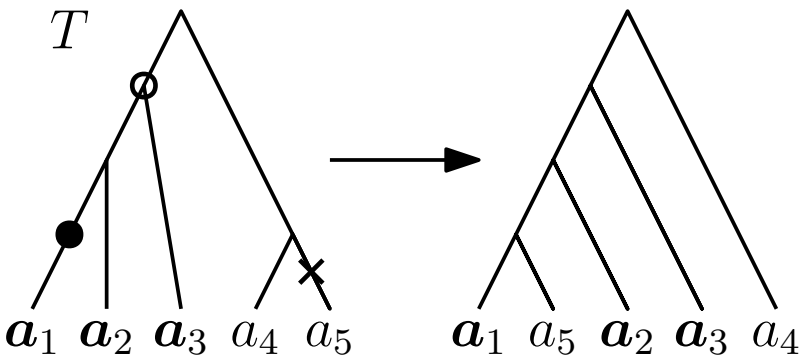


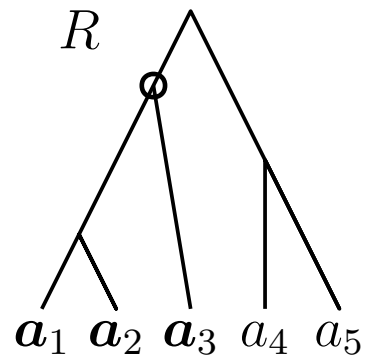


Shared cluster: $\{a_1, a_2, a_3\}$

Shortest paths

Clusters





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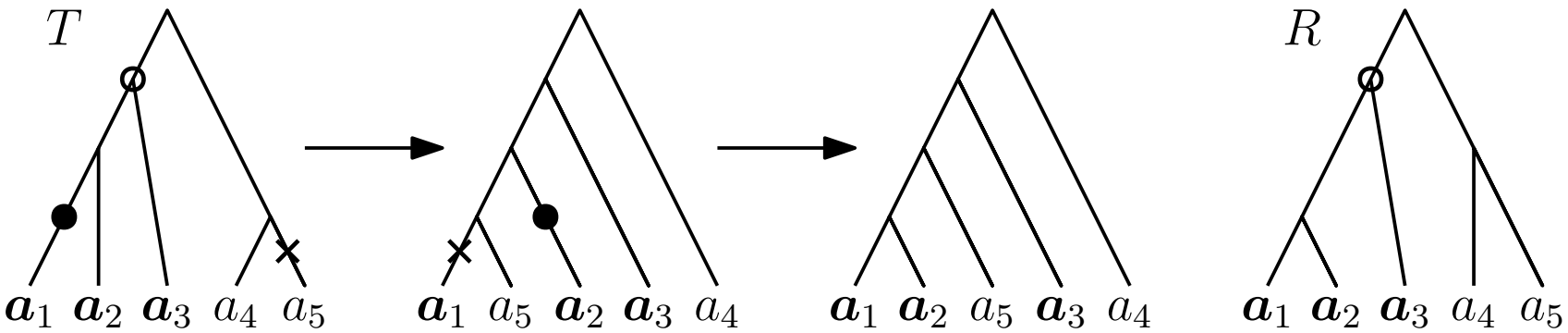

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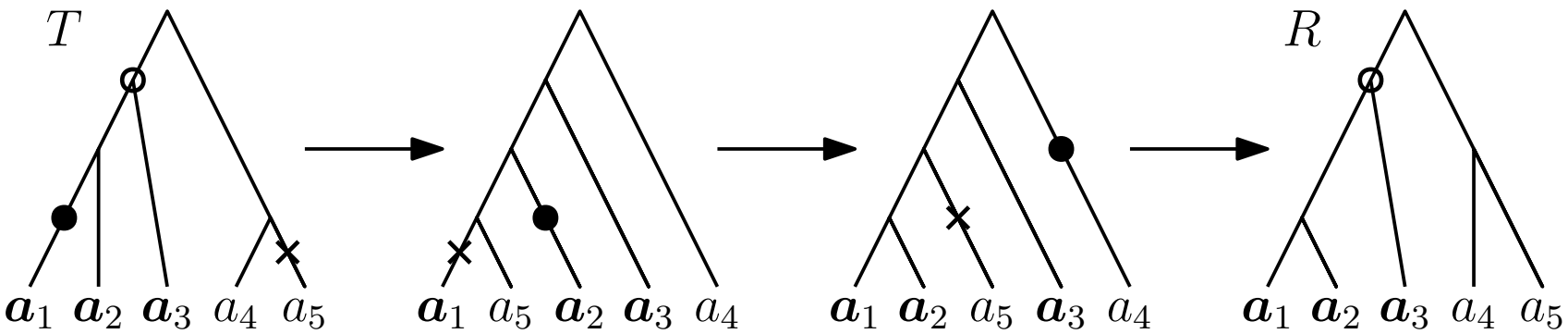

Shared cluster: $\{a_1, a_2, a_3\}$

Adding leaves

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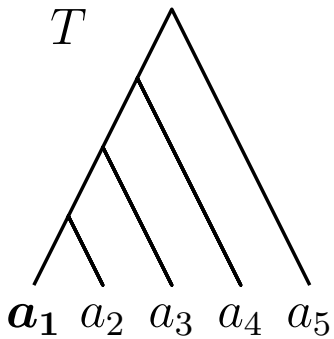

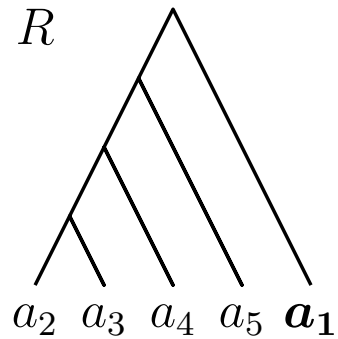

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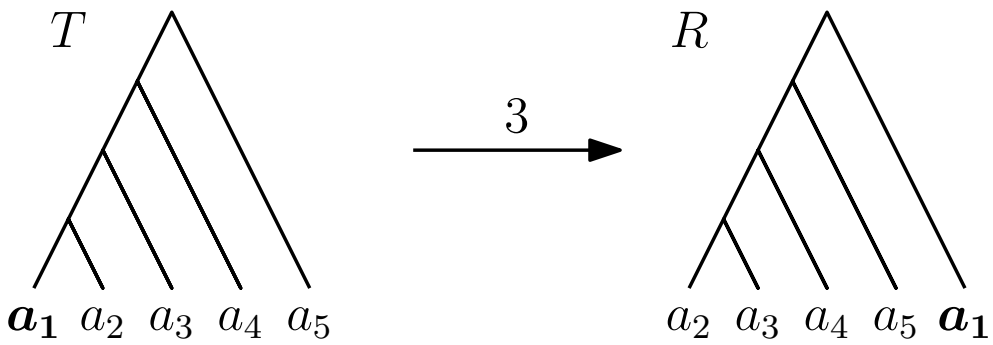

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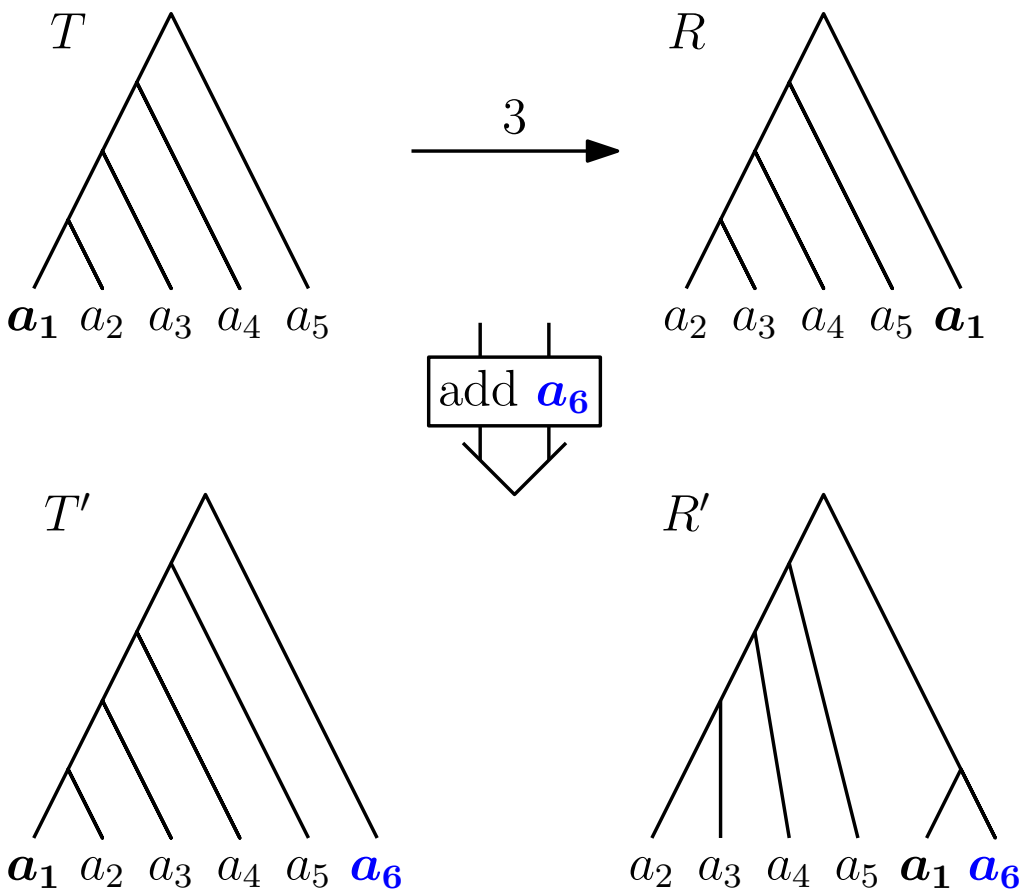

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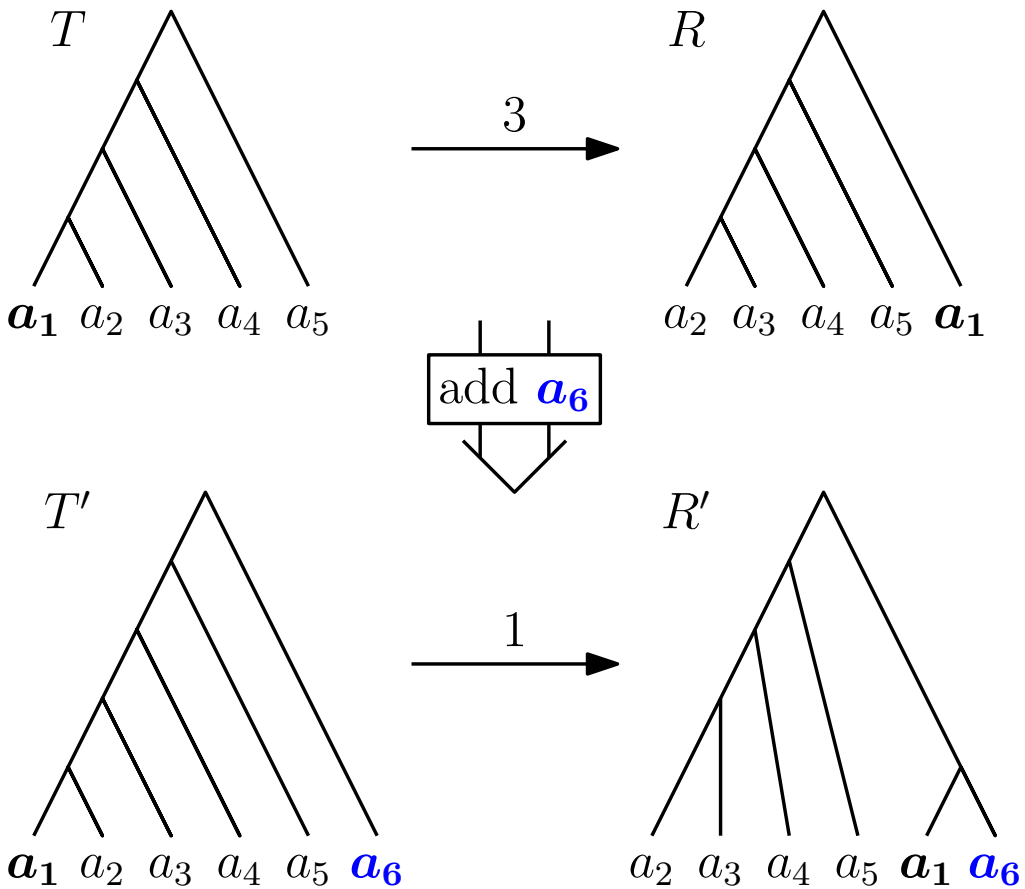

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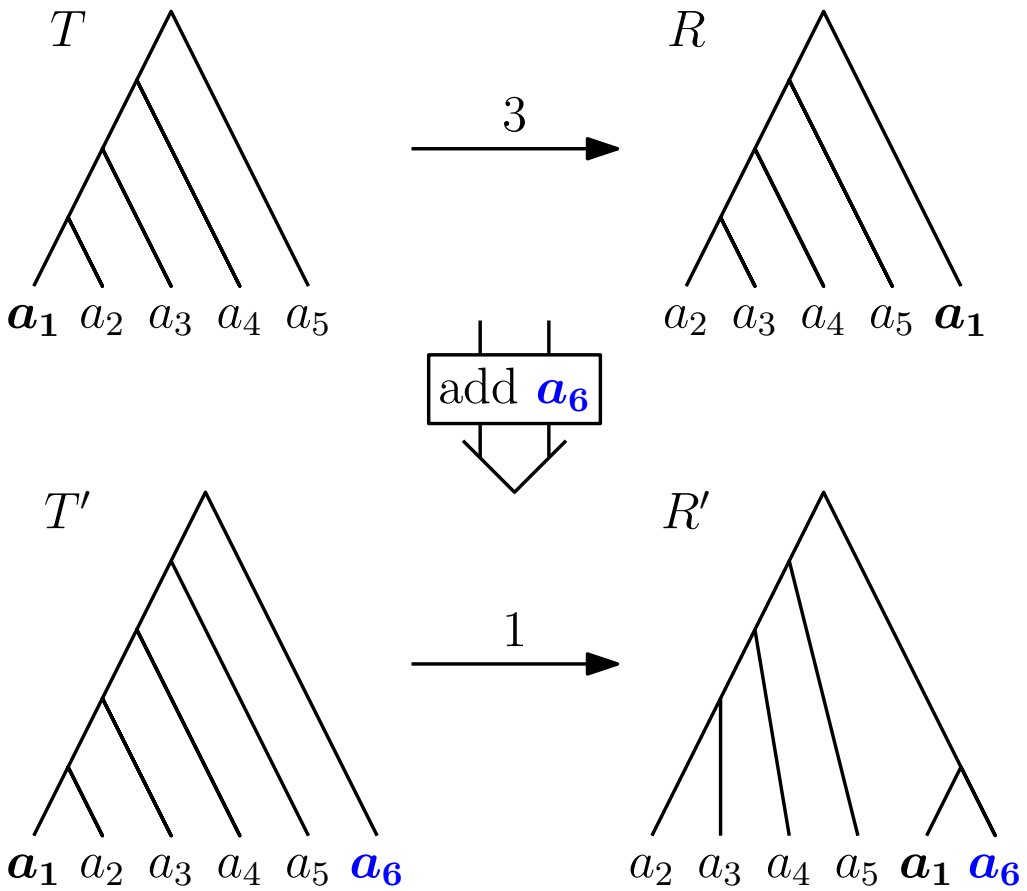

Theorem

Adding a leaf to a tree can decrease the distance by $O(n)$.

Thank you

Joint work with:

Alex Gavryushkin
(University of Canterbury, NZ)

Chris Whidden
(Dalhouise University, CA)